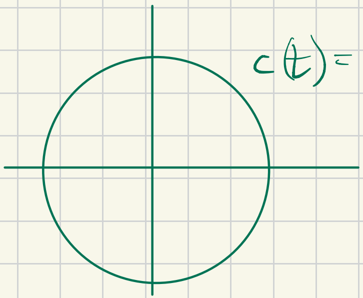
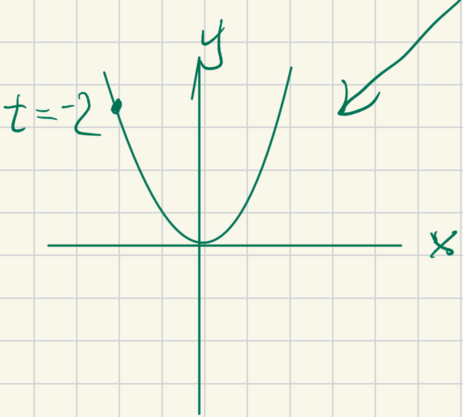
2.4 Paths and curves

We all know what curves are. Often we can describe them in more than one way:

- as points satisfying an equation, like $y = x^2$
- parametrically, like $c(t) = (t, t^2)$

$c(t) = (\cos t, \sin t)$





This section is about the parametric description. It provides a different way to describe functions $c : \mathbb{R} \to \mathbb{R}^n$, and also a way to understand the curves.

Terminology: a path is a mapping
$c : [a,b] \to \mathbb{R}^n$ $\quad [a,b] \subseteq \mathbb{R}$

The velocity vector of a path c(t).

If $c(t) = (c_1(t), \ldots, c_n(t))$ are the component functions, the velocity of c is
$c'(t) = (c_1'(t), \ldots, c_n'(t))$

It's length is the speed. It points in the tangent direction to the curve at $c(t)$.

Typical question: Consider the path
$c(t) = t(\cos(t), \sin(t))$
a. Find the velocity vector of c at $t=2$

$$c'(t) = \left(\cos t - t\sin t,\ \sin t + t\cos t\right)$$

b. Find the equation of the tangent line to the curve at $c(2)$.

$$v = c(2) + s\, c'(2)$$